KIT Department of Informatics
Institute for Anthropomatics and Robotics (IAR)
High Performance Humanoid Technologies (H^2T)

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Solution Sheet 5

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Solution 1 (Friction Triangles)

1. $\beta = arctan\, \mu = arctan\, 1 = \frac{\pi}{4} = 45°$
2. The friction triangles are shown in Figure 1.

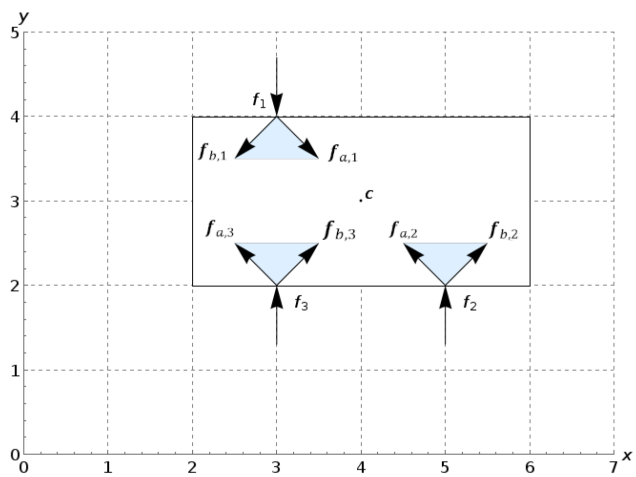


Figure 1: The friction triangles at the points $\mathbf{p_1}$, $\mathbf{p_2}$ and $\mathbf{p_3}$.

3. The force of friction $\mathbf{f^R}$ acts perpendicular to $\mathbf{f}$ with $|\mathbf{f_R}| = \mu|\mathbf{f}|$.
   The two force vectors at the edges of each friction triangle can be computed as follows: $\mathbf{f^a} = \mathbf{f} + \mathbf{f^R}$ and $\mathbf{f^b} = \mathbf{f} - \mathbf{f^R}$.

$$\mathbf{f_1^R} = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}, \quad \mathbf{f_2^R} = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}, \quad \mathbf{f_3^R} = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}.$$

   It follows:

$$\mathbf{f_1^a} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, \quad \mathbf{f_1^b} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$
$$\mathbf{f_2^a} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \quad \mathbf{f_2^b} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$
$$\mathbf{f_3^a} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \quad \mathbf{f_3^b} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}.$$

Solution 2 (Grasp Wrench Space)

1. In a first step, the vectors from the center of mass to the contact points are determined:

$$\mathbf{d_1} = (\mathbf{p_1} - \mathbf{c}) = (-1, 1)^\top,$$
$$\mathbf{d_2} = (\mathbf{p_2} - \mathbf{c}) = (1, -1)^\top,$$
$$\mathbf{d_3} = (\mathbf{p_3} - \mathbf{c}) = (-1, -1)^\top.$$

The corresponding moments can be calculated as follows:

$$\tau_1^a = \mathbf{d_1} \times \mathbf{f_1^a} = (-1, 1)^\top \times (0.5, -0.5)^\top = (-1 \cdot -0.5 - 1 \cdot 0.5) = 0,$$
$$\tau_1^b = \mathbf{d_1} \times \mathbf{f_1^b} = (-1, 1)^\top \times (-0.5, -0.5)^\top = (-1 \cdot -0.5 - 1 \cdot -0.5) = 1,$$

$$\tau_2^a = \mathbf{d_2} \times \mathbf{f_2^a} = (1, -1)^\top \times (-0.5, 0.5)^\top = (1 \cdot 0.5 - -1 \cdot -0.5) = 0,$$
$$\tau_2^b = \mathbf{d_2} \times \mathbf{f_2^b} = (1, -1)^\top \times (0.5, 0.5)^\top = (1 \cdot 0.5 - -1 \cdot 0.5) = 1,$$

$$\tau_3^a = \mathbf{d_3} \times \mathbf{f_3^a} = (-1, -1)^\top \times (-0.5, 0.5)^\top = (-1 \cdot 0.5 - -1 \cdot -0.5) = -1,$$
$$\tau_3^b = \mathbf{d_3} \times \mathbf{f_3^b} = (-1, -1)^\top \times (0.5, 0.5)^\top = (-1 \cdot 0.5 - -1 \cdot 0.5) = 0$$

The resulting *wrenches* are:

$$\mathbf{w_1^a} = (0.5, -0.5, 0)^\top,$$
$$\mathbf{w_1^b} = (-0.5, -0.5, 1)^\top,$$

$$\mathbf{w_2^a} = (-0.5, 0.5, 0)^\top,$$
$$\mathbf{w_2^b} = (0.5, 0.5, 1)^\top,$$

$$\mathbf{w_3^a} = (-0.5, 0.5, -1)^\top,$$
$$\mathbf{w_3^b} = (0.5, 0.5, 0)^\top.$$

2. GWS for the contact points $\mathbf{p_1}$ and $\mathbf{p_2}$:

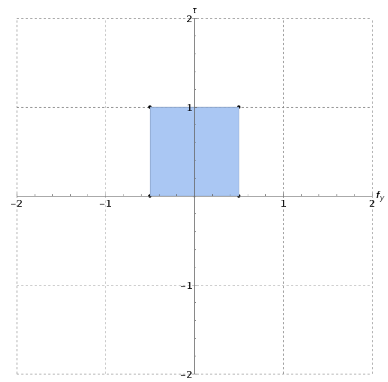


Figure 2: The convex hull of the *wrenches* at the points $\mathbf{p_1}$ and $\mathbf{p_2}$ (dimensions f_y and τ).

3. GWS for the contact points $\mathbf{p_1}$, $\mathbf{p_2}$ and $\mathbf{p_3}$:

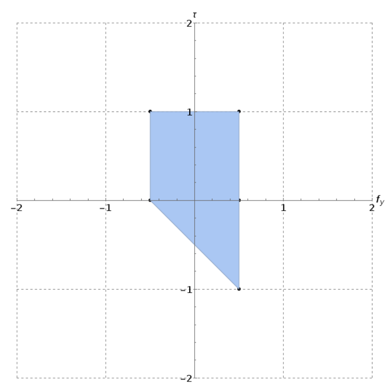


Figure 3: The convex hull of the *wrenches* at the points $\mathbf{p_1}$, $\mathbf{p_2}$ and $\mathbf{p_3}$ (dimensions f_y and τ).

Solution 3 (Force Closure)

1. The three-finger grasp is force-closed because the origin lies within the *Grasp Wrench Space* and the minimum distance to the edge is $\varepsilon > 0$.

2. The two-finger grasp is not force-closed because the minimum distance of the origin to the edge of the *Grasp Wrench Space* is $\varepsilon = 0$.

3. The ε-metric indicates the minimum distance of the origin to the edge of the convex hull of the *wrenches*. Procedure:

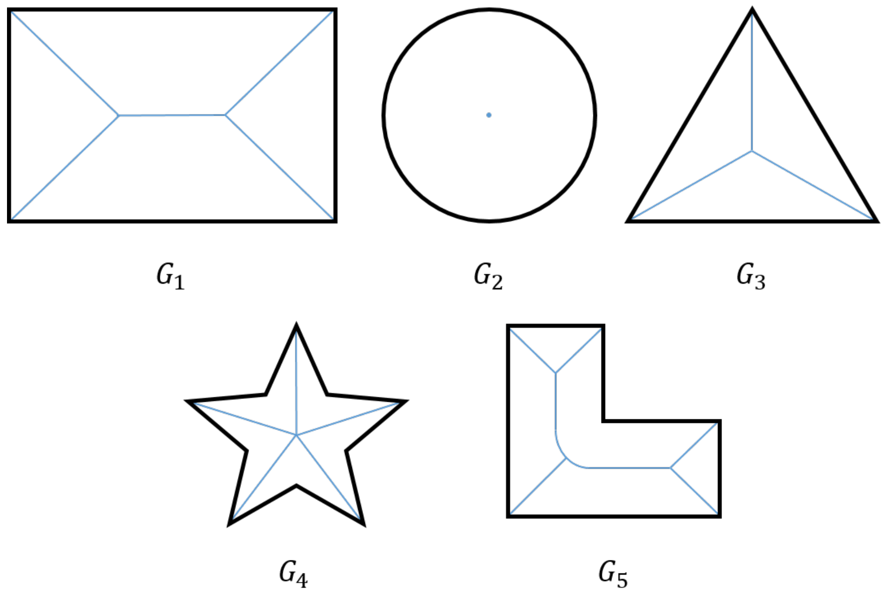
   - Determine the *wrenches* at the contact points.
   - Construct the *Grasp Wrench Space* as the convex hull of the *wrenches*.
   - Determine the minimum distance of the origin to the edge of the convex hull.

   The three-finger grasp has a ε-metric greater than zero ($\varepsilon > 0$).

   The two-finger grasp has a ε-metric of zero ($\varepsilon = 0$).

Solution 4 (Medial Axes)

The medial axes of the regions $G_1, \ldots, G_5$ are shown in the following sketch:



The principle of the medial axis can be visualized by drawing some of the maximum circles. Below an example is shown for region G_4: